# A Multiple Master based method for scaling glyphs without changing the stroke characteristics 

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This essay was originally written as part of the MA Typeface Design programme at the University of Reading in 2006. It was later published in the academic journals Document Numérique and Digital Creativity. The method described here has been implemented as Font Remix Tools, a set of plugins for FontLab and Glyphs.


#### Abstract

This article presents methods for handling Multiple Masters (several type styles such as regular and bold joined in one font file) beyond the conventional applications such as generating semibold fonts. The "boldness" information in an MM font can be used to change the size of glyphs without changing the stroke weight. The method helps to create small caps, Cyrillic lowercase and - since the horizontal and vertical scale factors can be chosen independently - even true condensed fonts.

Comparisons to existing typefaces with designed small caps and condensed versions show that the output generated by the model is rather close to that of the true glyphs.

In contrast to "intelligent" data formats that generate glyphs, the model suggested here is a technology-independent formula for the processing of pre-existing shapes. The output has the same format as the input and can be subsequently modified, making the method a tool for the automation of a specific design step rather than a stand-alone technology.


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## "True Small Caps Are More <br> Than Crushed Capitals

That Look Like They Had
Been Washed At Too High
A Temperature."
(Rögener et al., 1995:50, translated by the author)

1. Scaling changes the stroke weight

## 1. OBJECTIVES

Digital type technology has seen various innovations and advancements since its beginnings more than forty years ago. The progress is ongoing, with OpenType and the increasing support of Unicode being recent ground breaking improvements of the handling of glyphs. In the field of glyph shapes however, the development seems to have come to a halt years ago. The seventies, eighties and early nineties have seen more than 150 font formats (Karow 1994:22), various mathematical methods were developed and partly controversially discussed. After this phase of innovation and discussion TrueType and PostScript, both being outline formats, "have won the competition" (Jackowski 1999:118). After the abandonment of Adobe's Multiple Master technology it seems that static glyph shape descriptions, defined directly by the designer without the use of dynamic mathematics, are accepted by everyone as the only practicable system in professional print production. New approaches are neither dared by mathematicians nor desired by designers and typographers.

This essay resumes the discussion of mathematical models related to glyph shapes. In contrast to "intelligent" formats such as Metafont or Intellifont the model suggested here will not serve the creation of shapes. It is purely a tool for the processing of preexisting glyphs. The output has the same format as the input and can be subsequently processed, allowing for the manual correction of remaining deficiencies. This suggests the method to be used by typeface designers as a tool that automates a specific design step, supporting most designers' preference of "working out" as opposed to "thinking out" (Warde 1935), of making decisions as opposed to postponing them, like it would be typical for parametric design systems. However, further applications such as post-processing fonts by layout software or acting as a research tool for analysing type designs can be imagined.

Many glyphs used by typographers are derived from other glyphs and differ only in size. However, if glyphs are scaled the stroke weight changes accordingly, which leads to unacceptable alterations of the characteristics (Fig. 1).

If the new glyphs are not to be designed from scratch two ways to produce them are thinkable. One is to change the size by shifting portions of the outline instead of applying the same scaling factor to the coordinates of every point. This can be done manually (Hudson 2005) or with the help of an intelligent computer programme (André and Vatton 1994).

Another approach is to scale the glyph geometrically and correct the stroke weight. Usually this is done manually. As will be shown in this essay, it can also be done with the help of a mathematical model. Such a model will be developed and tested below. The stroke weight can be controlled with the help of a second master of a different weight that contains the necessary "boldness" information which can be added and subtracted as needed.

The Multiple Master technology provides a numeric description of the glyph outlines and allows us to easily process the two corresponding shapes. While being obsolete as a final font format it is widely used as a production tool and its support in font editors is still improving.

The model - which we will call "compensated scaling" - could be helpful to both experienced and less experienced designers. The former would

3. Anisotropic interpolation
be enabled to save time. Less experienced designers could even improve the design, as the consistency within a font or family would be improved. Just like a circular saw that cannot build furniture by itself and maybe does not even improve the final result of the master's work it can save a great deal of time. It performs certain aspects of the process very well and allows for subsequent refinement.

This essay is organised as follows. Section 2 presents the model. Starting with a simple principle, the formulae will be refined as various aspects are taken into account. Section 3 aims to test the model. In addition to comparing it directly to the Ek-module its output will be compared to existing, well-known typefaces, for small caps, condensed versions and cyrillic lowercase letters. The comparison will show whether the output of the mathematical model is close enough to that of the true one to be able to serve as an appropriate and time-saving basis for further refinement by the designer. Another aim of comparing the automatic and the true shapes is not a test of the creative potential of the method but a demonstration of its analytical use. Regarding the automatically scaled glyphs as stylistically unchanged variations of their base the comparison with manually designed versions reveals modifications that the designers in question may have done unconsciously. Section 4 discusses in detail different applications of the method. Section 5 points out some singular properties of the approach and draws conclusions of the study.

## 2. THE METHOD

If a glyph is scaled down the stoke gets thinner, too. With the help of a second, bolder master it is possible to increase the weight. The method developed here will not be a process or algorithm but only one equation for $x$ and $y$ coordinates each, which processes the shapes in a single step. However, as a means of exemplification some steps will be explained as if they were performed in a certain sequence.

The compensation for the loss in stroke weight is notionally done before the scaling. This means that an intermediate instance - an interpolation between regular and bold - is scaled (Fig 2). The required instance can be calculated.

Condensed versions can be created by applying only the horizontal changes, which is called anisotropic interpolation. The $x$ value of each point is that of the scaled semibold while the $y$ values remain the same as in the regular (Fig 3).

## A first simple approach

The assumption is that a regular weight glyph should be scaled down by the factor $\mathbf{s}$ while the loss in stroke weight is compensated for.

This is achieved by scaling an interpolation between the regular master $\left(\mathbf{x}_{\mathbf{r}}, \mathbf{y}_{\mathbf{r}}\right)$ and the bold master $\left(\mathbf{x}_{\mathbf{b}}, \mathbf{y}_{\mathbf{b}}\right)$, represented by the interpolation factor $\mathbf{q}$.

$$
\begin{aligned}
& y=s\left[q y_{r}+(1-q) y_{b}\right] \\
& x=s\left[q x_{r}+(1-q) x_{b}\right]
\end{aligned}
$$

[1]

The factor $\mathbf{q}$ can be determined by calculating the stroke weight of the final result. The typical stroke weight of the regular may be $\mathbf{r}$ and the stroke weight of the bold master $\mathbf{b} \cdot \mathbf{r}$ with $\mathbf{b}$ as the quotient of the two weights. As the weight needs to remain unchanged the original stroke weight $\mathbf{r}$ is equated with the final stroke weight.

$$
\begin{equation*}
r=s[q r+(1-q) b r] \tag{2}
\end{equation*}
$$

The equations can be solved for $\mathbf{q}$

$$
\begin{equation*}
q=(1 / s-b) /(1-b) \tag{3}
\end{equation*}
$$

The stroke width $\mathbf{r}$ can be cancelled straight away, which shows that the model does not rely on strokes nor does it try to recognise them. The notion of stroke weights is only a temporary means of exemplification. Only b, the relation between the typical bold and regular weight, is processed.

The formulae [1] and [3] define the final shape. They could be united into one equation, which is not necessary here. These formulae do not depend on Bézier splines, they can be applied to any format as long as there are two corresponding sets of data.

## More specific control over the stroke weight

Often a complete compensation of the stroke weight is not desired. For example, small caps are typically slightly lighter than capitals. Therefore a stroke scale value a is introduced: it is between o for a full compensation as described above and 1 for no compensation, i.e. pure geometric scaling of the regular shape. The value $\mathbf{a}$ is defined in a way that the stroke weight is actually scaled by the factor $s^{a} . s^{0}=1$ means that the stroke weight does not change and $\mathrm{s}^{1}=\mathrm{s}$ means that the stroke is scaled in the same way as the overall shape is scaled. Equations [2] and [3] change accordingly:

$$
\begin{aligned}
& s^{\mathrm{a}} r=s[q r+(1-q) b r] \\
& q=\left(s^{a-1}-b\right) /(1-b)
\end{aligned}
$$

This abstract definition of the parameter allows the designer to choose the value intuitively with some experience. Furthermore, if the scale value is adjusted - e.g. while trying out different sizes of small caps - the stroke scale value can typically remain unchanged.

## Anisotropic scaling

In most cases horizontal and vertical scaling needs a different factor. The mathematical solution is very simple: $\mathbf{s}_{\mathbf{x}}$ and $\mathbf{s}_{\mathbf{y}}$ are independent factors for scaling in $x$ and $y$ direction and we need independent values $\mathbf{q}_{\mathbf{x}}$ and $\mathbf{q}_{\mathbf{y}}$ for horizontal and vertical interpolation:

$$
\begin{align*}
& y=s_{y}\left[q_{y} y_{r}+\left(1-q_{y}\right) y_{b}\right]  \tag{1b}\\
& x=s_{x}\left[q_{x} x_{r}+\left(1-q_{x}\right) x_{b}\right] \\
& q_{y}=\left(s_{y}^{a-1}-b\right) /(1-b)  \tag{3b}\\
& q_{x}=\left(s_{x}^{a-1}-b\right) /(1-b)
\end{align*}
$$

## Taking into account italic angles

If the glyph is scaled anisotropically the italic angle changes. In order to avoid this effect the glyph is slanted left before scaling and back after scaling (Fig 4).

$$
d \rightarrow d \rightarrow d \rightarrow d
$$

## 4. Notional order of processing italics

The order of these steps is only notional; in fact they are performed at the same time, in one equation:

$$
\begin{align*}
& y=s_{y}\left[q_{y} y_{r}+\left(1-q_{y}\right) y_{b}\right]  \tag{1c}\\
& x=s_{x}\left[q_{x}\left(x_{r}-y_{r} i\right)+\left(1-q_{x}\right)\left(x_{b}-y_{b} i\right)\right]+y i
\end{align*}
$$

With $\mathbf{i}$ being the tangent of the italic angle, in formula [1b], $\mathbf{x}_{\mathbf{r}}$ and $\mathbf{x}_{\mathbf{b}}$ are modified by subtracting $\mathbf{y}_{\mathbf{r}} \mathbf{i}$ and $\mathbf{r y}_{\mathbf{b}} \mathbf{i}$ respectively, representing the slant to the left. The term yi slants the glyph back to its original angle.

This results in a slight slant of the Bézier tangents that typically define a glyph. Depending on the technology this can be an issue. It can be tackled with appropriate methods but it is not related to the mathematical problems discussed here.

If a change in italic angle is desired this can be achieved by adjusting the value used as $\mathbf{i}$.

## Optical sizes

If there is an optical size axis in the font we have specific control over hairlines as well as main stems. Instead of processing two masters there are four versions, each with a typical hairline and main stem weight. The coordinates of the four masters may be $\left(\mathbf{x}_{\mathbf{r}}, \mathbf{y}_{\mathbf{r}}\right)$ for the regular, $\left(\mathbf{x}_{b}, \mathbf{y}_{\mathbf{b}}\right)$ for the bold, ( $\mathbf{x}_{\mathrm{rd}}, \mathbf{y}_{\mathrm{rd}}$ ) for regular display and ( $\mathbf{x}_{\mathrm{bd}}, \mathbf{y}_{\mathrm{bd}}$ ) for bold display. The interpolation factors are $\mathbf{q}$ for the weight axis and $\mathbf{p}$ for the optical size:

$$
\begin{align*}
y= & s_{y}\left[p_{y}\left(q_{y} y_{r d}+\left(1-q_{y}\right) y_{b d}\right)+\left(1-p_{y}\right)\left(q_{y} y_{r}+\left(1-q_{y}\right) y_{b}\right)\right]  \tag{1d}\\
x= & s_{x}\left[p_{x}\left(q_{x}\left(x_{r d}-y_{r d} i\right)+\left(1-q_{x}\right)\left(x_{b d}-y_{b d} i\right)\right)+\right. \\
& \left.+\left(1-p_{x}\right)\left(q_{x}\left(x_{r}-y_{r} i\right)+\left(1-q_{x}\right)\left(x_{b}-y_{b} i\right)\right)\right]+y i
\end{align*}
$$

The determination of $\mathbf{p}$ and $\mathbf{q}$ is done with an equation similar to [2a]. For the main stems, the typical stroke weight of the regular may be $\mathbf{R}$ and the stroke weight of the bold master $\mathbf{B} \cdot \mathbf{R}$ with $\mathbf{B}$ as the quotient of the two weights. For the hairlines, the typical stroke weight of the regular may be $\mathbf{r}$ and the stroke weight of the bold master $\mathbf{b} \cdot \mathbf{r}$.

The main and hairline widths are calculated as follows:

$$
\begin{align*}
& s^{a} R=s\left[p\left(q R_{d}+(1-q) B_{d} R_{d}\right)+(1-p)(q R+(1-q) B R)\right]  \tag{2d}\\
& s^{a} r=s\left[p\left(q r_{d}+(1-q) b_{d} r_{d}\right)+(1-p)(q r+(1-q) b r)\right]
\end{align*}
$$

The solution of these two equations with the two unknowns $\mathbf{p}$ and $\mathbf{q}$ requires only basic mathematics. However, the final equations become very long so they shall not be discussed here. $\mathbf{p}$ and $\mathbf{q}$ can easily be calculated by a computer.

## Scaling the bold master and enlarging

To keep it simple, the assumption so far has been that the lighter of the two masters is to be scaled and the size of the glyph is decreased. However, the formulae do not need any modifications to work with other settings. If the bold master is scaled we pretend it was the "regular" one and the other masters is still defined as $\mathbf{b} \cdot \mathbf{r}$ with the "boldness" factor $\mathbf{b}<\mathbf{1}$ this time. In [3] this leads to an interpolation factor $\mathbf{q}>\mathbf{1}$, which means extrapolation.

If the lighter master is enlarged, meaning $\mathbf{s}>\mathbf{1}$, then $\mathbf{q}$ becomes negative, which again means extrapolation. If the bold master is enlarged this leads to interpolation with $\mathbf{0}<\mathbf{q}<\mathbf{1}$.

## Spacing and Kerning

The left sidebearing is not calculated specifically - it is equal to the $x$-value of the left-most node. The advance width of the glyph is based on the advance widths of the two input masters with the same kind of equation as [1b] or [1d]. This leads to harmonic and coherent spacing as can be exemplified with a thought experiment (Fig. 5).

# Hamburgefons <br> Hamburgefons 

5. Derivation of spacing and kerning

The letters of any word are added together into a single glyph, as is sometimes done for logotypes. The borders between the letters and therefore the advance widths are stored as virtual markers. The whole word is then scaled. If we assume that the scaling method produces sensible, correct proportions within glyphs, we can then extract the correct sidebearings from the resulting multi-letter glyph. The same is true for kerning, which can also be calculated with an equation equivalent to [1b] or [1d]. Only kerning within the set of scaled glyphs can be calculated this way, unlike the kerning between the scaled and unscaled glyphs.

An alternative way to deal with sidebearings is to keep them the same as they were before the scaling.

## Notes on the actual scale factor

Regarding the advance width and the total dimensions the glyph is typically not scaled by s. This is only the case if these values are the same in all involved masters. The deviation from the nominal factor $\mathbf{s}$ even varies from glyph to glyph. However, as the desired scale factor is usually found by trial and error this lack of control over the final output is not a problem. On the contrary, the fact that the output contains some of the other master's proportions leads to a more coherent result than scaling factors forced individually for each glyph. This will be exemplified in the comparison with the Ek-module later on.

The height of small caps can be controlled specifically as the height of the capitals is typically the same in all masters.

relation of the handle lengths

kink due to incompatible handles

6a. Requirements regarding handles at the same node

## Equalising the input

In order to avoid kinks in the resulting outline, if the corresponding tangents of the input masters are not parallel then the ratio of the length of the "handles", the lines connecting the nodes to the adjacent control points, must be equal (Adobe 1995:14, see Fig. 6a). It can be added that the same is necessary for nodes that connect a curve and a line - the ratio between the length of the line and the handle must be equal (Fig. $6 b)$. These requirements can be derived from the theorem of intersecting lines.

In addition to this the lengths of the handles within one curve segment should be comparable in all masters (Fig. 7). Considering that the $y$-value of any point of the bold master is completely discarded when a condensed font is created it is easy to see that the relation of the handle lengths has a significant influence on the final result.

The exact definition of "comparable" could be different to the one given in Figure 7. The equalisation programme we have written for this purpose actually uses a slightly refined definition. The mentioned requirements might overdetermine the handles, in which case the equality within a curve segment is given less priority. Due to lack of space the algorithm that equalises the curves will not be discussed here.

7. Requirements regarding handles within a curve

One might wonder whether it is possible to change the control points without altering the shape of the glyphs. In fact, Bézier curves imply one redundant degree of freedom so changing the length of one handle can be compensated for with the help of the other handle in the curve, virtually without any visible effect on the shape. It should be emphasised here that redundancies are the single most important source of any problems related to interpolation and extrapolation, be it isotropic or anisotropic. While the redundancy related to Bézier curves is inevitable and can be tackled with a macro programme, it is the designer's responsibility to set up an appropriate structure of nodes that avoids further redundancies.

## Implementation

The method is implemented as a Python macro for FontLab. The dialogue box demonstrates the necessary input (Fig. 8). The fields for the typical stem widths are automatically filled in on the basis of the standard stems in the global hinting parameters of the font. If these are set cor-

| RMX Scaler Easy |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Wto |  | Wt1 |  |
| standard stem | 80 |  | 220 |  |
| width | 85.0 | \% | 90.0 | \% |
| height | 80.0 | \% | 85.0 | \% |
| keep stroke | 90.0 | \% | 65.0 | \% |
| adjust spacing | 30 |  | 20 |  |
| vertical shift | 0 |  | 0 |  |
| add suffix | .sc |  |  |  |
| V preview |  |  |  |  |
|  |  | ance |  | K |

8. User interface for two masters
rectly they do not need to be changed. The programme only uses the quotient of the stems b so it usually does not matter whether the stems of the minuscules or the majuscules are referred to.

The values for horizontal and vertical scale factors and stroke scale are obviously to be chosen by the user, so is the master or the masters the macro is applied to. The tangent of the italic angle needs to be entered manually.

If "keep sidebearings" is checked the sidebearings are not affected by the process, which is useful when modifications to single glyphs are made during the design process. When a set of glyphs or a whole font is transformed the option should be unchecked in order to obtain coherent spacing. It should be added that in many cases subsequent global adjustments to the letterspacing need to be made using the standard tools of the font editor.

The transformation performed by the macro could be simulated by a series of actions using the built-in features of FontLab. However, this would lead to rounding errors and take a great deal of time, cancelling the main advantage - saving time.

## 3. TEST OF THE METHOD

In order to find out how close the output of the method is to a final version and to see what kinds of refinement are necessary small caps, condensed versions and cyrillic lowercase characters have been generated automatically for existing typefaces. Appendices B to F show comparisons between the original and automatically generated versions. In each case two versions of the original fonts were assembled in an MMfont and then equalised with a macro programme as described above. The necessary parameters were chosen so that the result is as close as possible to the original but no manual changes were made.

In order to make the differences more apparent they were amplified with the help of extrapolation. For example, triple original and triple auto means that the difference was tripled. In other words, in a multiple master setup the four lines represent the positions -2000, 0, 1000 and 3000. The exaggerations in both directions looks wrong and inharmonic. They are not meant for judging visual qualities but only as a rational means of recognising the differences.

## Small caps: Meta

This exaggeration shows some of the particular properties of the real small caps. The optical corrections in A and $v$ that prevent black spots at the joint of the strokes are more pronounced in the small caps. It seems that for the small caps great care was taken to produce an even grey value. Especially the bold $\mathrm{m}, \mathrm{N}$ and w are significantly too heavy in the automatic version (Fig. 9), which becomes apparent in the text setting shown in the appendix. One could argue if this is the fault of the method that created them. A closer look at the bold and black capital letters shows that this problem is already present in the input of the macro.

9. Comparison of Meta original and automatically generated small caps

The reason for this issue - which was neglected by the designer consciously or unconsciously in the capitals but not in the small caps - is described by Paul Renner (1939:25, translated by the author) as "subjective irradiation": "A stroke width deviation almost incapable of measurement could be more disturbing in smaller sizes (...) than in larger ones; often it is detected only then because it manifests itself as a deviation not so much of width (...) but of brightness. (...) It is especially disturbing where two strokes form an acute angle or intersect. Here, in the $M$ W K N and so on tiny zones form that are not sufficiently brightened by the irradiation; as spots they disturb the overall appearance of many typefaces - especially in small sizes - so much that they cannot be used in small caps setting."

## on on on on on on on on on on on on

10. Myriad, Frutiger 57 and 87 (top to bottom),
double original, original condensed, automatic version and double auto (left to right)

## Condensed fonts: Frutiger and Myriad

In all condensed fonts created the most obvious difference is an increased "angularity", a tendency towards hyperelliptic curves in the original versions. It can be seen in Myriad as well as Frutiger in both weights (Fig. 10). In Myriad this comes along with a tendency towards more open counters in $\mathrm{C}, \mathrm{G}, \mathrm{S}, \mathrm{a}, \mathrm{c}, \mathrm{e}, \mathrm{g}$ and s .

The diagonal of the $z$ has a noticeably wrong weight. As can be seen in the other appendices this is a general problem with diagonals and shall therefore be explained in detail. In each master, the weight of the diagonal, a one-dimensional value, is controlled by two degrees of freedom (Fig. 11), resembling a redundancy. Since any vertical information from the bold master is discarded when the condensed is generated this twodimensional interplay gets lost, which can lead to inadequate diagonals. The dotted line indicates a thinkable alternative for the lower left two nodes. Although the weight of the diagonal in the bold master would not change it would lead to a lighter diagonal in the condensed because the resulting nodes move to the right while the changed height of the upper node is ignored.

11. Node structure of the $z$ in Myriad regular and bold

Another redundancy which seems to play a major role in the $z$ of Myriad is given in the position of the node that sits on the diagonal. Within a certain range it could be shifted along the diagonal without visibly changing the shape, as indicated by R. In the bold $z$ the relative positions of the considered node within the diagonal is further right and, after the anisotropic interpolation, too far right in the final result, making the diagonal too heavy.

Where round strokes join vertical stems, such as in $n$ and $p$, the reduction in weight is exaggerated in the automatic version. Presumably the reason for this inadequate weight is similar to the one described for the $z$ but the interplay of nodes and control points is more complex here.

## ouvmasc ouvmasc ouvmasc

13. Frutiger 87, the automatic condensed version and Frutiger 75

Some glyphs show different relative widths in the manually designed version. Compensated scaling applies the width proportions of a certain semibold instance, preserving horizontal proportions. Frutiger made the $s$ and the c relatively wider in the regular weight while in Frutiger 87, the $o$ is unproportionally narrow (Fig. 12 and 13). This is clearly a design decision which shows that, although the macro has obviously worked as intended the design of a condensed font requires additional decisions that can not be provided by an automatism.

(Karow 1994:110)
14. Instructions in the Ikarus system

## megon megon

15. Leipziger Antiqua, condensed by the Ek-module (top), and by compensated scaling

## Cyrillic lowercase: Gauge and Den Haag

Many cyrillic letters share the same basic shape in their uppercase and lowercase versions. This makes it interesting for the use of compensated scaling. The relevant lowercase letters were generated automatically, which does not necessarily reflect the order in which the letters would be designed. In case the minuscules letters were designed before the majuscules these would be created by enlarging the lowercase.

As shown in appendix E, after an initial attempt to create the letters with global scale factors it became clear that the relative proportions in the lowercase differ from the uppercase. Zhukov (1996:11) states that "the visual order of Cyrillic lower-case is quite different from the one governing the forms of Latin-script minuscule. (...) Nuances in proportion and weight, and details of construction, play very important roles in this visual order." He presents some quantitative analysis of the cyrillic alphabet to describe its peculiarities. However, his taximetrics do not take into account proportion or weight.

In order to create the closest possible result different horizontal scale factors were applied to the individual letters. This "key" gives some insight into the structure of cyrillic. The test with Gauge showed that the key is not universal but further research may find general principles depending on the typeface style. Once the required key has been established it will be easier to create cyrillic typefaces, especially for nonnative designers. The lack of sense of the correct proportions could be compensated for with the help of this rational approach.

While the length of the vertical serifs needs significant corrections, the vertical ones come very close to the designed ones. This is owed to the optical size axis and formal structure of this particular font and might require further corrections in other cases.

## Comparison with the Ek-module

The $E k$-module is a part of the hz-programme developed by urw in Hamburg. It condenses and expands glyphs without the need of a second master (Karsh 1993:7). This is only possible through the use of additional information stored with the outline data:
"Since a definition of uprights and curves has now been incorporated into the programmes, curves and stem widths now remain unaltered by changes in setting. Under extreme conditions curves would also indicate an irregular change in shape but programme modifications have been incorporated to offset this and now permit automatic curve adjustments." (Anonymous 1981:10) Obviously the additional instructional input (Fig. 14) is used by the programme to distort the outlines in a way that preserves stroke weights. This could be interpreted as "shifting portions of the outline" as mentioned in the introduction.

The comparison of $E k$-module and compensated scaling as shown in appendix $G$ shows how similar the results are despite a very different approach. Most differences are due to the slightly different initial unscaled shapes. In the narrowest version however, the widths of certain letters differ (Fig. 15). Compensated scaling adds a certain amount of the bold master and with it the bold proportions, as described earlier.

## 4. APPLICATIONS

The previous section implied some possible uses for compensated scaling. These and other applications shall be discussed in a more methodic way.

## Generating new glyphs

A typeface family typically consists of several weights with several hundred glyphs each. As there are many regularities within it describing and then applying these regularities can significantly reduce the number of glyphs that need to be designed from scratch (Fig. 16). As shown in Appendix $A$, a large number of glyphs can be composed of other glyphs without any modifications to the shape. Accented characters require a small amount of design input, namely the placement of the diacritics in relation to the base glyph. Regularities within the relative positions allow for a further reduction of the required human input with the help of so-called anchors. Rotating and shifting can create further glyphs based on others. More than half of the glyphs in a font can typically be generated in this way.

Compensated scaling reduces the number of glyphs that need to be designed even further. Taking Minion as an example, 172 or $39 \%$ of the remaining 442 glyphs could be generated automatically, at least as a basis for further refinement.

It could be asked whether the use of compensated scaling has an effect on the final shape of the glyphs, even if the designer makes all the corrections he considers necessary. If the tool has an effect on the design at all we can assume that the result is more consistent with the existing glyphs, which might be sensed as "too perfect" or "boring" by some. However, there is no rational argument that could put the maxim of greatest possible visual consistency into question. Extensions to existing typefaces by someone other than the original designer are a special though not uncommon case when the introduction of new design input has to be kept at a minimum.

## Width corrections

An application of the method that does not create new glyphs is the correction of the widths of individual letters. The fine-tuning of the proportions consumes a considerable portion of the time spent on a type design. Manfred Klein (1991:132 et seqq.) describes the development of Poppl Pontifex in 1973-74, still on an analogue basis then, as "a long road, with many design and correction hazards". Changing the widths of letters played an important role: "Berthold's experts, led by Günter Gerhard Lange, advised him, 'Tighter, more classical, smoother'. (...) At G.G. Lange's suggestion, letters such as $\mathrm{h}, \mathrm{g}, \mathrm{m}, \mathrm{n}$ and o were now somewhat narrower". In the next design step changes in letter widths were again necessary. "Seven lower-case letters and two upper-case letters had to be narrowed still further." Later, he "drew thirty-eight letters narrower without surrendering the character of the style he was after." We can assume that with the digital design tools available today this long road would have been shorter or at least more comfortable. Compensated scaling provides a further - and drastic - speed-up of the design process, reducing the act of making glyphs narrower or wider to a single step.

## Adjustments of global proportions

Compensated scaling is capable of adjusting the x-height or globally adjusting the width of the font, or the width of capitals as compared to the minuscules. Due to the reasons explained in the introduction to do this manually would be fraught with considerable effort. If the x-height is changed with the help of compensated scaling then only the length of the ascenders and descenders need to be corrected afterwards, which takes only a fraction of the time necessary using the conventional approach.

## Post-processing fonts

Outside the production of fonts the model could be applied in text composition playing the same role as the Ek-module in the hz-programme, which condenses and expands letters to aid justification and to produce an even grey value of the text. Compensated scaling could also be implemented in a system proposed by Peter Karow (1998:279): "Extensions to a type manager. The method for expanding and condensing as well as the methods for optical scaling can be implemented into a type manager. It would then be possible to load an MM-font consisting of two weights only, and still be able to make the typical three dimensional interpolation," in which he refers to the weight, width and optical size axes. The method described here could not provide the optical size interpolation, however.

In contrast to using compensating scaling as a design tool this implementation would allow the end user to freely choose the scaling factors. However, the advantage over static, specifically defined size or width variants supplied by the typeface designer is debatable. A disadvantage similar to the use of Multiple Master technology on an end user basis is that the result could not be corrected for formal deficiencies anymore.

## Analysing existing typeface designs

From an analytical point of view, the test of the compensated scaling model has produced, as a by-product, some hints on the "typical" differences between the true glyphs and the output of the method. Although they are not gained or presented in a systematic way we have already learned various things that could be applied in the design of a new typeface. They give an impression of what the tool could be used for in a methodic approach.

To enquire into this matter is to ask the following, very fundamental questions: What makes a true condensed apart from the fact that the characters are generally narrower? What makes cyrillic lowercase and small caps different from the capital letters apart from the fact that they are smaller and slightly wider?

If we compare capitals and small caps directly the stylistic differences can hardly be recognised, because the difference in size is visually dominant and overpowers the more subtle aspects. Eliminating the size differences by generating stylistically unchanged versions with compensated scaling makes it possible to see and rationally discuss things that are not visible in a direct comparison. Systematic research in this field would require an analysis of many more typefaces than we have used for the purpose of this essay - a rewarding realm of future typographic research.

17. Construction, processing and reproduction offonts

## 5. CONCLUSIONS

Describing aesthetics and ideal shapes with the help of mathematics has been a challenge for some of the best artists for centuries. Constructions of ideal shapes have been proposed for many arts, including type design - with compass and ruler (Carter 1991) or with the help of computer programs (Knuth 1982). As in our writing system the glyphs are supposed to work as a set and show no individual peculiarities a global description that generates the alphabet seems sensible.

For technical, aesthetic and pragmatic reasons recognising and describing the regularities within a font provides an attraction. Reducing the formal content to a minimum, even forcing equality and consistency, is adequate to the fundamentals of writing. This is exactly what was done when handwriting, with its almost identical letters, was transposed to printing with its forced identity of shapes.

However, in the field of letters mathematical equality is not the same as visual equality. In order to look the same many elements of glyphs may not be identical. Numerous dimensions in details and the overall shape need to be tweaked in individual glyphs. For example, a bold font can not be created with a global mathematical concept. One can not globally modify the width of the strokes that make up the letters - let alone shapes that can not be interpreted as strokes.

Outline formats are certainly the "winner" of the disputes about glyph representation methods because they do not rely on stems or strokes, providing freedom to the designer.

The same is true for the method of compensated scaling, which was developed and tested in this essay. Although it requires two masters with different stroke weights it relies entirely on the designer to provide input that fulfils this requirement. The explicit definition of boldness for each shape is both the designer's obligation - and freedom. By treating every point of the outline in exactly the same way compensated scaling does not spoil the essence of the design. It preserves all the characteristics and tweaks made by the designer, recombining two manually designed shapes.

Obviously, even if all necessary adjustments and modifications are accounted for in the input, new issues arise when the size or proportions of glyphs are changed, which became apparent in the test of the method.

However, our test has shown that the output generated by the model is rather close to that of the true glyphs. Even if the extent of this closeness was not quantified we suggest that it is great enough for the model to be a very appropriate basis for further refinement. Above all, "appropriate" means that compensated scaling can greatly help the designer save work.

As a mathematical tool for the processing and combining of shapes compensated scaling stands between the theories for the construction and the technologies for the reproduction of glyphs that are designed on the basis of the designer's visual judgements (Fig. 17). It integrates the "two cultures" as described by Carter (1991), overcoming "that dichotomy [which] is deep in our habits of thought and education - if not in the actual anatomy of the right and left hemispheres of the brain, as it is fashionable to think."

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Overwiew of glyph set

## 270 Glyphs need to be designed from scratch:

 I J K L M N O P Q R S T U V W X Y Z [ $\wedge$ _







## 172 Glyphs are scaled versions of existing ones:




```
$ - ( ) h l 0 2 2 3 4 4 5 6 7 7 8 % 9 % % 2 % 8 \ \mathbb{C}
```



```
? A B C D E F G H I J K L M N O P Q R S T T U V W X Y Z i & & Ø P
```


## 785 Glyphs can be composed of existing shapes:

 Ú UU Ü Ý à á â ã ä å æç è é ê ë ì í î ï ñ ò ó ô õ ö ù ú $\hat{\text { û }}$ ü ý ÿ $\overline{\mathrm{A}}$ ā $\overline{\mathrm{A}}$

 ’n N y Ō ō Ǒ ŏ Ơ ő R ŕr










" " $\%$ く > / 0
 ffk fft fh fj fk ft fb fh fi fk fl ff ffi fll \#

 ( ) [ ] \{ \} i $\dot{i}$ " »く > - - .

## ABCDEFGHIJKIMNOPQRSTUVWXYZ triple original

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

original small caps

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

automatic small caps

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> triple



Meta with original small caps
Lerind ofort escir end corebe af a vilien ale ate jores, glie piers ing un Havers med un verste to iscaul jes vis de latun pura une de nar comige, atto dis FANCHIM ARE BERM HEM, inge bould onichie cor offen cond hanten compars miate Unince com ineven hen tre er atesseli nottua sicoran hammie, quardes astalst he, ders dem enva rog, nuare sampond, sin tre der par inta fugge, oform dingesolicti, ment ter elache luist annesche porders, desender lustratie tolume obier. Estion sombonio note mine faciast voirot ver sinonosa per orriken, lorancon Cort dide jecher Somet in digende strop ducauser grasse nums ran der hemparno van isige, rist dig enn mes ten estans seigetter das son, CONDE TANS FORT ING OCOR turden Forcied der inde als de sinangel Cordammor
automatically generated small caps
Lerind ofort escir end corebe af a vilien ale ate jores, glie piers ing un Havers med un verste to iscaul jes vis de latun pura une de nar comige, atto dis FANCHIM ARE BERM HEM, inge bould onichie cor offen cond hanten compars miate Unince com ineven hen tre er atesseli nottua sicoran hammie, quardes astalst he, ders dem enva rog, nuare sampond, sin tre der par inta fugge, OFORM DINGESOLICTI, ment ter elache luist annesche porders, desender lustratie tolume obier. Estion sombonio note mine faciast voirot ver sinonosa per orriken, lorancon Cort dide jecher SOMET IN DIGENDE strop ducauser grasse nums ran der hemparno van isige, rist dig enn mes ten estans seigetter das son, CONDE TANS FORT ING OCOR turden Forcied der inde als de sinangel Cordammor

# ABCDEFGHIJKLMNOPQRSTUVWXYZ tiple original 

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

original
small caps

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> triple <br> auto

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> Meta Black capitals <br> ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> Meta Bold capitals

Meta with original small caps

Lerind ofort escir end corebe af a vilien ale ate jores, glie piers ing un Havers med un verste to iscaul jes vis de latun pura une de nar comige, atto dis FANCHIM ARE BERM HEM, inge bould onichie cor offen cond hanten compars miate Unince com ineven hen tre er atesseli nottua sicoran hammie, quardes astalst he, ders dem enva rog, nuare sampond, sin tre der par inta fugge, OFORM DINGESOLICTI, ment ter elache luist annesche porders, desender lustratie tolume obier. Estion sombonio note mine faciast voirot ver sinonosa per orriken, lorancon Cort dide jecher somet in digende strop ducauser grasse nums ran der hemparno van isige, rist dig enn mes ten estans seigetter das son, CONDE TANS FORT OCOR turden Forcied der inde als de sinangel
automatically generated small caps

Lerind ofort escir end corebe af a vilien ale ate jores, glie piers ing un Havers med un Verste to iscaul jes vis de latun pura une de nar comige, atto dis FANCHIM ARE BERM HEM, inge bould onichie cor offen cond hanten compars miate Unince com ineven hen tre er atesseli nottua sicoran hammie, quardes astalst he, ders dem enva rog, nuare sampond, sin tre der par inta fugge, OFORM DINGESOLICTI, ment ter elache luist annesche porders, desender lustratie tolume obier. Estion sombonio note mine faciast voirot ver sinonosa per orriken, lorancon Cort dide jecher Somet in digende strop ducauser grasse nums ran der hemparno van isige, rist dig enn mes ten estans seigetter das son, CONDE TANS FORT OCOR turden Forcied der inde als de sinangel

## abcdefghijklmnopqrstuvwxyz abcdefghijklmnopqrstuvwxyz abcdefghijklmnopqrstuvwxyz <br> Myriad condensed abcdefghijklmnopqrstuvwxyz <br> double auto

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> double original

Myriad condensed

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> automatic version <br> ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> double auto

## Myriad Condensed

Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber stageter-sarion lezar rave comberepres int ofambrine hosigtemen Hersters te tan, tue Tragentaffe conach stel, sumede lens pas. Eliento porezalosse trach so venten volle. Altel accon pon tent irem

Automatic version
Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber stageter-sarion lezar rave comberepres int ofambrine hosigtemen Hersters te tan, tue Tragentaffe conach stel, sumede lens pas. Eliento porezalosse trach so venten volle. Altel accon pon tent irem

## abcdefghijklmnopqrstuvwxyz $\substack{\text { dombe } \\ \text { oingmed }}$ abcdefghijkImnopqrstuvwxyz abcdefghijkImnopqrstuvwxyz acemanic abcdefghijklmnopqrstuvwxyz $\begin{gathered}\text { doubl } \\ \text { aute }\end{gathered}$

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

double original

ABCDEFGHIJKLMNOPQRSTUVWXYZ
original Frutiger 57

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

automatic version

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

## Frutiger 57

Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber stageter-sarion lezar rave comberepres int ofambrine hosigtemen Hersters te tan, tue Tragentaffe conach stel, sumede lens pas. Eliento porezalosse

## Automatic version

Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber stageter-sarion lezar rave comberepres int ofambrine hosigtemen Hersters te tan, tue Tragentaffe conach stel, sumede lens pas. Eliento porezalosse

## abcdefghijklmnopqrstuvwxyz

double original

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abcdefghijkImnopqrstuvwxyz
abcdefghijkImnopqrstuvwxyz
original
Frutiger 87
automatic version

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

## double

 auto
## ABCDEFGHIJKLMNOPQRSTUVWXYZ

original Frutiger 87

## ABCDEFGHIJKLMNOPQRSTUVWXYZ

automatic version

## ABCDEFGHIJKLMNOPQRSTUVWXYZ <br> double <br> auto

## Frutiger 87

Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber

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Sattimor demorrisen son her den nicarel inand hissen Ver hie perporecuppens ondrun tossis hil son offesie men hen gathor Lande Sache diet ore den pre vor dinen beschelsancre, munthendie rept lache furdine lairen, hetimmen undessam ber atitiverche comoniez andas ande atierebil puni fortigine aircarg sie ho, breschesen us sie nos seltinza pas. Era en yable Tusige leine lagelle, nongenist den le sentestace donsce auchallier ton, man tre dige, trispen, des not no suang enas dierente, handied incor sit frastivervie man sinessanber

## APPENDIX E

## Den Haag Pro (Alexander Tarbeev, 1998)

Simulation of some cyrillic lowercase letters.
Although a Multiple Master width axis was avilable these are based only on a regular and bold version.

# ВГДжЗИКЛмнПтцчшщъыьэюя ВГДЖЗИКЛМНПТЦЧШЩЪЫьэЮя <br> original lowercase <br> <br> ВГДЖЗИКЛМНПТЦЧШЩЪЫьЭЮя <br> <br> ВГДЖЗИКЛМНПТЦЧШЩЪЫьЭЮя <br> automatic lowercase <br> <br> вГДЖЗИКЛМНПТЦЧШШЪЫьЭЮЯ <br> <br> вГДЖЗИКЛМНПТЦЧШШЪЫьЭЮЯ <br> <br> triple <br> <br> triple <br> <br> auto 

 <br> <br> auto}

## ВГДЖЗИКЛМНПТЦЧШЩЪЫьЭЮЯ <br> original capitals

## ВгдЖЗИКЛМНПтЦЧШЩЪЫЬЭЮя

## Den Haag

илучаюте иса кого ковое решени избежа или по илизадей изион. Комии объем, фицарав цельне арее террази ее учным инага питакой. На помет и пешает охолюче и ная обя ещегдае их обой целяет сивает иза обсть учальши, а фикакой обязациод обрее ожносим Пикако оду обы и ле свертию очене, вымию фиции илишь вающие ость подухода, ее цию объекта объясь испонна отрерга. Подожены деских уссех о получис пиростив можитере и ее авитемируем и и его Ваго и деятие инции и а кой этоку онии иможе воимого тельзове объявид, пологоде объекти и опечесающая ная ости опрой, аниможет осве ил поначеся и зацию соземы уг из эти изаций, главля какимичести. Коможе изую виях, ногдатери запрафиз недеятерет зация

## Automatic version

илучаюте иса кого ковое решени избежа или по илизадей изион. Комии объем, фицарав цельне арее террази ее учным инага питакой. На помет и пешает охолюче и ная обя ещегдае их обой целяет сивает иза обсть учальши, а фикакой обязациод обрее ожносим Пикако оду обы и ле свертию очене, вымию фиции илишь вающие ость подухода, ее цию объекта объясь испонна отрерга. Подожены деских уссех о получис пиростив можитере и ее авитемируем и и его Ваго и деятие инции и а кой этоку онии иможе воимого тельзове объявид, пологоде объекти и опечесающая ная ости опрой, аниможет осве ил поначеся и зацию соземы уг из эти изаций, главля какимичести. Коможе изую виях, ногдатери запрафиз недеятерет зация

## APPENDIX E

## Den Haag Pro (Alexander Tarbeev, 1998)

Simulation of some cyrillic lowercase letters. In this second version the horizontal scale factor was adjusted individually for each letter. o and c were not generated automatically.

Scale factors ( $1,0.7329$ ), stroke scale 0.12 then scale factors ( $s_{x}, 1$ ), stroke scale 0.18 , spacing +8/1000 em each side

## ВГДЖЗИКЛМНПТЦЧШЩЪЫЬЭЮЯ <br> triple original <br> ВГДЖЗИКЛМНПТЦЧШЩЪЫЬЭЮЯ <br> original lowercase

## ВГДЖЗИКЛМНПТЦЧШЩЪЫЬЭЮЯ

automatic
lowercase

## ВГДЖЗИКЛМНПТЦЧШШЪЫЬЭЮЯ


$S_{x}$

## Den Haag

Илучаюте иса кого ковое решени избежа или по илизадей изион. Комии объем, фицарав цельне арее террази ее учным инага питакой. На помет и пешает охолюче и ная обя ещегдае их обой целяет сивает иза обсть учальши, а фикакой обязациод обрее ожносим Пикако оду обы и ле свертию очене, вымию фиции илишь вающие ость подухода, ее цию объекта объясь испонна отрерга. Подожены деских уссех о получис пиростив можитере и ее авитемируем и и его Ваго и деятие инции и а кой этоку онии иможе воимого тельзове объявид, пологоде объекти и опечесающая ная ости опрой, аниможет осве ил поначеся и зацию соземы уг из эти изаций, главля какимичести. Коможе изую виях, ногдатери запрафиз недеятерет зация

Automatic version

Илучаюте иса кого ковое решени избежа или по илизадей изион. Комии объем, фицарав цельне арее террази ее учным инага питакой. На помет и пешает охолюче и ная обя ещегдае их обой целяет сивает иза обсть учальши, а фикакой обязациод обрее ожносим Пикако оду обы и ле свертию очене, вымию фиции илишь вающие ость подухода, ее цию объекта объясь испонна отрерга. Подожены деских уссех о получис пиростив можитере и ее авитемируем и и его Ваго и деятие инции и а кой этоку онии иможе воимого тельзове объявид, пологоде объекти и опечесающая ная ости опрой, аниможет осве ил поначеся и зацию соземы уг из эти изаций, главля какимичести. Коможе изую виях, ногдатери запрафиз недеятерет зация

# ВГДЖ-ИКЛМНОПТХЦЧШШЪЫЬ-ЮЯ 

ВГДЖЗИКЛМНОПТХЦЧШЩЪЫЬЭЮя
triple original
original lowercase
automatic lowercase

## ВГДЖЗИКЛМНОПТХЦЧШЩЪЫЬЭЮЯ

## ВГДЖーИКЛМНОПТХЦЧШШЪыь-Юя



Gauge regular

Илучаюте иса кого ковое решени избежа или по илизадей изион. Комии объем, фицарав цельне арее террази ее учным инага питакой. На помет и пешает охолюче и ная обя ещегдае их обой целяет сивает иза обсть учальши, а фикакой обязациод обрее ожносим Пикако оду обы и ле свертию очене, вымию фиции илишь вающие ость подухода, ее цию объекта объясь испонна отрерга. Подожены деских уссех о получис пиростив можитере и ее авитемируем и и его Ваго и деятие инции и а кой этоку онии иможе воимого тельзове объявид, пологоде объекти и опечесающая ная ости опрой, аниможет осве ил поначеся и

Automatic version

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Simulation of the condensing and expanding performed by the Ek-module, demonstrated with the Leipziger Antiqua (Albert Kapr, 1971).

The upper line shows the result of the Ek-module (Anonymous 1981:10), the lower line is based on a digitisation of the Leipziger Antiqua and a bold version designed by the author.

# Hamburgefons Hamburgefons 

scaled by 0.84 , full compensation, spacing unchanged

## Hamburgefons Hamburgefons

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## Hamburgefons Hamburgefons

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